

Unit 1: Algebra & Number (Part A)

Part A consists of 3 outcomes that need to be covered:

✓ **AN1: Demonstrate an understanding of factors of whole numbers** by determining the prime factors; greatest common factor; least common multiple; square root; and cube root.

✓ **AN2: Demonstrate an understanding of irrational numbers** by representing, identifying, simplifying and ordering irrational numbers.

AN3: Demonstrate an understanding of powers with integral and rational components.

Outcome AN3 will be covered in Sections 4.4, 4.5, and 4.6. Today, we will look at the following:

Express powers with rational exponents as radicals and vice versa, when m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m \quad \text{and} \quad \left(x^{\frac{m}{n}}\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Activate Prior Learning:

Exponent Laws

Product of powers law: $a^m \cdot a^n = a^{m+n}$

Quotient of powers law: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power law: $(a^m)^n = a^{mn}$

Write as a single power.

a) $3^2 \cdot 3^5$

b) $(4^2)^5$

c) $(-5)^{10} \div (-5)^8$

$$x^2 \cdot x^3 = x^5$$

$$\frac{x^5}{x^2} = x^3$$

$$(x^2)^3 = x^6$$



4.4 Fractional Exponents and Radicals

LESSON FOCUS

Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?



Use a calculator to complete the table.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

Use a calculator to complete the table.

x	$x^{\frac{1}{3}}$
1	
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	
125	

$$\frac{1}{3} = 0.\bar{3}$$

$$0.333333$$

4.4 Fractional Exponents and Radicals

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$a^{\frac{2}{3}} = \left(\sqrt[3]{a} \right)^2$$

index

Example #1

$$8^{\frac{2}{3}} = \left(\sqrt[3]{8} \right)^2 = (2)^2 = 4$$

#2

$$\left(\sqrt[3]{70} \right)^3 = 70^{\frac{3}{3}}$$

$$\left(\sqrt[6]{666} \right)^6$$

$$= 666^{\frac{6}{6}} = 666^1 = 666$$

$$\begin{aligned}
 \left(\frac{27}{8}\right)^{\frac{2}{3}} &= \left(\sqrt[3]{\frac{27}{8}}\right)^2 \\
 &= \left(\frac{3}{2}\right)^2 \\
 &= \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} &= 5^{\frac{1}{2} + \frac{1}{2}} \quad \text{and} \quad \sqrt{5} \cdot \sqrt{5} = \sqrt{25} \\
 &= 5^1 & &= 5 \\
 &= 5
 \end{aligned}$$

$$\underbrace{\sqrt[3]{\left(\frac{3}{8}\right)^4}}_{\text{denominator}} = \left(\frac{3}{8}\right)^{4/3}$$

numerator

$$\left(\frac{27}{8}\right)^{2/3} =$$

$$= \frac{9}{4} = 2\frac{1}{4}$$

Example 1

Evaluating Powers of the Form $a^{1/n}$

Evaluate each power without using a calculator.

- a) $27^{1/3}$ b) $0.49^{1/2}$ c) $(-64)^{1/3}$ d) $\left(\frac{4}{9}\right)^{1/2}$

 **SOLUTION**



CHECK YOUR UNDERSTANDING



$$40^{\frac{2}{3}} = \left(\sqrt[3]{40} \right)^2$$

$$40^{\left(2 \right)^{\left(\frac{1}{3} \right)}}$$

- a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.
b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.

SOLUTION

- a) Use $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$.

$$40^{\frac{2}{3}} = (\sqrt[3]{40})^2 \text{ or } \sqrt[3]{40^2}$$

Example 1**Evaluating Powers of the Form $a^{\frac{1}{n}}$**

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

SOLUTION

The denominator of the exponent is the index of the radical.

a) $27^{\frac{1}{3}} = \sqrt[3]{27}$
 $= 3$

b) $0.49^{\frac{1}{2}} = \sqrt{0.49}$
 $= 0.7$

c) $(-64)^{\frac{1}{3}} = \sqrt[3]{-64}$
 $= -4$

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}}$
 $= \frac{2}{3}$



CHECK YOUR UNDERSTANDING



4.4 Fractional Exponents and Radicals

Example 2**Rewriting Powers in Radical and Exponent Form**

- a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.
 b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.

SOLUTION

a) Use $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$.
 $40^{\frac{2}{3}} = (\sqrt[3]{40})^2$ or $\sqrt[3]{40^2}$

b) Use $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

$$\sqrt{3^5} = 3^{\frac{5}{2}}$$

The index of the radical is 2.

Use $(\sqrt[n]{a})^m = a^{\frac{m}{n}}$.

$$(\sqrt[3]{25})^2 = 25^{\frac{2}{3}}$$



CHECK YOUR UNDERSTANDING



4.4 Fractional Exponents and Radicals

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#7

a) $8^0 = 1$

b) $8^{1/3} = \sqrt[3]{8} = 2$

c) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

d) $8^{3/3} = 8^1 = 8$

e) $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$

f) $8^{5/3} = (\sqrt[3]{8})^5 = 2^5 = 32$

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3, 4, 5, 6, 8, 10, 11, 12, 15

Monday, Oct.3

- Review fractional exponents and class work from Friday
- Begin Section 4.5 (negative exponents)
- Notes/Examples
- Classwork/Homework

Reminder: Unit 1 Test on Thursday!!! Extra help available at lunch time.

Classwork/Homework

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